Benha University
Faculty of Engineering – Shoubra
Mechanical Engineering Department

كلية الهندسة بشبرا

Engineering Mathematics (1) A
Final Exam. 1/1/2018
Time allowed 3 hour

First year (production Branch)

Total Mark (105) 21 for each question

Answer the following questions:

Question (1)

(a) Use the power series to solve the differential equation $y'' - 2xy' + y = 0$.

(b) Given $w = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$

Show that $(w_x)^2 + (w_y)^2 = (w_r)^2 + \frac{1}{r^2}(w_\theta)^2$

(c) Find the local extrema of the function $f(x, y) = x^2 + 4y^2 - x + 2y$

Question (2)

(a) If $\phi = 2x^3y^2z^4$ find $\vec{\nabla}.\vec{\nabla}\phi$. and $\vec{\nabla}\times\vec{\nabla}\phi$

(b) Evaluate $\int_C \vec{F}.d\vec{r}$ where $\vec{F} = (5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}$ along the curve

$y = x^3$ from the point (1,1) to the point (2,8).

(c) Evaluate $\int_0^2 \int_{x^2}^{2x} (x^3 + 4y)\,dy\,dx$

Question (3)

Find the general solution of the following differential equations:

(a) $xydx + (x^2 + 1)dy = 0$

(b) $(3x + y)dx + (x + 3y)dy = 0$

(c) $y' + y.\tan x = \sin x$

Question (4)

(a) Find the general solution for $y'' - 5y' - 6y = e^x \sinh 6x$

(b) By variation of parameter solve $y'' + n^2 = \operatorname{cosec} nx$.

(c) Find the general solution for $xy'' + (x - 1)y' - y = 0$ given that $y = e^{-x}$ is one solution.

إنتبه : السؤال الخامس فى ظهر الورقة

Question (5)

(a) Evaluate $\oint_C \vec{F}.d\vec{r}$ where $\vec{F} = (2x - 3y)\,\vec{i} + y\,\vec{j}$ and C is the circle $x^2 + y^2 = 4$.

(b) Apply Green's theorem to evaluate $\oint_C \vec{F}.d\vec{r}$ where $\vec{F} = (x - y)\,\vec{i} + (x + y)\,\vec{j}$ and C is the closed curve in xy – plane consisting of $y = x^2, x = y^2$.

(c) Use the divergence theorem to evaluate $\oiint_S \vec{F}.\vec{n}ds$

where $\vec{F} = 2xy\,\vec{i} + y\,z^2\,\vec{j} + xz\,\vec{k}$ and S is the surface of parallelogram bounded by $x = 0, y = 0, z = 0, x = 2, y = 1, z = 3.$.

Dr. Fathi Abdessalam